Reply to Millis *et al.*: In a recent paper [1] we showed the equivalence of two seemingly contradictory theories [2, 3] on Griffiths-McCoy singularities (GMS) in metallic antiferromagnets close to a quantum critical point (QCP). Two generic features of these theories are the presence of ordered magnetic droplets in the paramagnetic phase, and a non-universal crossover from GMS to superparamagnetism below a temperature $T^* = \omega_0 \exp\{-C_2\}$, where ω_0 is a cut-off energy and C_2 is a non-universal constant dependent on the electronic damping of the spin excitations. In a recent comment, Millis *et al.* [4] argue that in heavy-fermion materials the electronic damping is large leading to the freezing of locally magnetically ordered droplets at high temperatures. In this reply we show that this conclusion is based on a treatment of the problem of disorder close to a QCP which is not self-consistent. We argue that a self-consistent treatment of the ordered droplets must lead to weak damping and to a large region of GMS behavior, in agreement with the results of ref. [2].

Clean heavy-fermion materials can be described by the Kondo lattice model: $H = H_0 + \sum_i J_K \mathbf{S}_i \cdot \mathbf{s}_i$, where $H_0 = \sum_{\mathbf{k},\sigma} \epsilon_{\mathbf{k}} c^\dagger_{\mathbf{k},\sigma} c_{\mathbf{k},\sigma}$ is the conduction electron Hamiltonian ($c^\dagger_{\mathbf{k},\sigma}$ and $c_{\mathbf{k},\sigma}$ are the creation and annihilation operators for electrons with momentum $\mathbf{k}$, spin σ, and energy $\epsilon_{\mathbf{k}}$), J_K is the Kondo coupling at site $\mathbf{R}_i$ between localized spins $\mathbf{S}_i$, and conduction electron spin $s^a_i = (\hbar/2) \sum_{\alpha,\beta} c^\dagger_{i,\alpha} \sigma^a_{\alpha,\beta} c_{i,\beta}$ ($\sigma^a_{\alpha,\beta}$ with $a = x, y, z$ are Pauli matrices). In the absence of a full theory of the heavy-fermion state close to a QCP we follow the argument proposed by Doniach [5] and recent results from extended dynamical mean-field theory (EDMFT) [6, 7]. The Kondo coupling is responsible for two competing effects that originate a QCP: the RKKY interaction, responsible for magnetic ordering, with characteristic energy scale $T_c \sim g^2 E_F$, and the Kondo effect, responsible for the heavy-fermion paramagnetic phase below a temperature $T_K \sim E_F \exp\{-1/g\}$, where E_F is the Fermi energy. The dimensionless coupling $g \sim J_K N(0)$, where $N(0)$ is the electronic density of states, determines the phase diagram at $T = 0$: when $g < g_c$ the system is magnetically ordered and the Kondo effect is suppressed; when $g > g_c$ Kondo singlets are formed and the system is paramagnetic. EDMFT calculations [6, 7] find that g_c is order of unit ($\sim \mathcal{O}(1)$). Inside the magnetic ordered phase, $g < g_c$, the system is an ordinary Fermi liquid with $N_{g<g_c}(0) \propto 1/E_F$ [7]. In the paramagnetic phase, $g > g_c$, the Kondo effect leads to a large renormalization of the density of states, $N_{g>g_c}(0) \propto 1/T_K$, and to a large effective mass, m^*: $N_{g>g_c}(0) \propto m^*_{g>g_c} \gg N_{g<g_c}(0) \propto m^*_{g<g_c}$, since $E_F \gg T_K$.

As we have shown in ref. [2], *in the presence of disorder* (either by modification of the electronic degrees of freedom via $N(0)$ or by the change in the magnetic interactions via J_K) the dimensionless coupling becomes position dependent, $g(i)$, and is therefore statistically distributed. Even if the material is in the paramagnetic phase there will be regions where locally $g(i) < g_c$ leading to the formation of magnetically ordered droplets of size N. It was also shown [2] that for antiferromagnetic droplets there is a critical droplet size $N_c \propto 1/g^2$ such that for $N > N_c$ the droplets freeze at low temperatures due to the electronic damping [9].

Ref. [3] finds that the density of states within the droplet is given by $N_{g>g_c}(0)$ so that $g(i) \approx J_K/T_K \sim \mathcal{O}(1)$, implying that $N_c \sim \mathcal{O}(1)$ and therefore concluding that essentially most droplets are frozen. This argument, however, is incorrect. Firstly, to have an ordered droplet in first place one has to require that locally $g(i) < g_c \sim \mathcal{O}(1)$. Since ref. [4] assume that $g(i) \sim \mathcal{O}(1)$ the formation of droplets is suppressed, in other words, if the local coupling is too large, leading to large dissipation as argued in ref. [4], the RKKY interaction is not able to stabilize local order and droplet formation. Secondly, and even more importantly, a droplet is a piece of the magnetically ordered phase and therefore the damping should be given by the *local* density of states, $N_{g(i)<g_c}(0)$, which is not enhanced. In this case, as argued in ref. [2], one has to use $N_{g<g_c}(0)$ for the droplet damping. Hence, $g(i) \approx J_K/E_F \ll 1 < g_c \sim \mathcal{O}(1)$, consistent with $N_c \gg 1$ and with the existence of GMS.

One can track down the inconsistency in the arguments of refs. [3, 4] to the inappropriate use of Hertz theory [8]. In Hertz approach one works with the order parameter alone and the effects of dissipation originate on the trace, in the partition function, of the electronic degrees of freedom. The trace is carried out *perturbatively* in g and leads to Landau damping with a damping rate, $\Gamma(\omega_n) \propto g^2 |\omega_n|$. Hence, it is questionable that this approach can be used to describe situations where $g \sim \mathcal{O}(1)$, and therefore, outside the applicability of perturbation theory. In the presence of droplets the trace over electrons has do be done self-consistently because the electronic properties are directly affected by the presence of local order in the system. In ref. [3] it was found that the local changes in the electronic properties are not significant. In that theory disorder enters only through the order parameter and in the generation of the droplets. Thus, there is no reason to believe that the dissipation rate is tied to a single energy scale $E_0 \sim T_K$ as proposed in ref. [4]. In ref. [3] the dissipation rate should be seen as free parameter of the theory and T^* can be obtained from the experimental data. We should stress that the microscopic theory of ref. [2] estimates that $N_c \approx 10^5$ for systems like $UCu_{4-x}Pd_x$. This result supports the conclusion that GMS can be the source of non-Fermi liquid behavior observed in a large class of U and Ce intermetallics [10].

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